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EFFECTS OF FOG DROPLETS ON WAKE VORTEX DECAY RATE

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16. ABSTRACT The report discusses a simple model for the motion of particles in a laminar line vortex. It is assumed that the particles are small, so that a Stokes drag is operative. The energy required to accelerate a set of these particles is determined and shown to be only a small fraction of the energy content of the vortex flow. It is shown that this energy transfer is unlikely to be sufficient to significantly modify the vortex decay rate. It is further argued that the effect of the particle on the viscous properties of the resulting two-phase fluid would lead to a slower decay rate than would occur in single-phase air flow. However, this conclusion may not necessarily follow for turbulence flows. Results of the study show that the migration of particles to the outer flow results in a redistribution of the velocity profile in the vortex and in a non-uniform two-phase viscosity across the core. Although not studied, it is suggested that these effects may accelerate vortex bursting.					
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FOREWORD

This report explores the effect of fog and cloud droplets on aircraft wake vortex decay rate. The question was originally posed by Dr. Vern Rossow of the NASA Ames Research Center. The report is exploratory and should be considered preliminary.

The research was conducted by Dr. T. H. Moulden and Dr. Walter Frost at the University of Tennessee Space Institute under the technical direction of Dr. George H. Fichtl of the NASA Marshall Space Flight Center. Resources for the conduct of the effort were made available by Mr. John Enders of the Aeronautical Operating Systems Division, Office of Advanced Research and Technology, NASA Headquarters.

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Notations

a	particle radius
c	wing chord
d/dx	an operation from the differential calculus
e (or \exp)	the function defined by $de(x)/dx = e(x)$
g	gravitational acceleration
i	quantity with the property $i^2 = -1$
k, k_1	constants
m	particle mass
n	index
t	time
u	particle velocity
u, v, w	velocity components along coordinate directions z, r, θ
w_o	tangential velocity immediately behind wing
z, r, θ	cylindrical coordinates
A_n	coefficient in Fourier expansion
F_z, F_r, F_θ	Force components along (z, r, θ)
J_ν	Bessel function of order ν
R	specific value for the vortex radius
Re	Reynolds number
r_h	half radius of vortex
U, V, W	velocity components of vortex along (z, r, θ)
U_∞	free-stream velocity
U_o	initial axial velocity of vortex
x	mass fraction of water
α	wing incidence
μ	fluid viscosity
μ_n	zero of Bessel function
π	quantity such that $e(\pi i) = -1$; or $\pi = 3.1416\dots$
ρ	density of fluid
ρ_w	density of particle
Γ	circulation of vortex
Φ	potential function
ψ	stream function
γ	kinematic viscosity
x	mass fraction of liquid phase
F_d	Froude number

Subscripts

e	effective value
g	gas
h	half-radius of vortex
max	maximum value
o	initial value at $t = 0$
TP	two-phase
w	liquid

I. Introduction

It has been suggested that the trailing vortex system behind an aircraft may decay more rapidly when fog is present in the atmosphere. Such an effect is of significance to airport scheduling where the hazards to aircraft operation associated with the strong vortices generated by the current class of large jet aircraft require time consuming separation distances between successive landings and take-offs. As airports become more congested, this situation creates flight delays and financial penalties. Thus, there is ample need to study all possible conditions under which the rate of vortex decay is increased.

The mechanism by which fog droplets would alter the rate of vortex decay is not fully understood. One can envision that the acceleration of the water droplets as they first enter the vortex would remove energy from the flow and retard its circumferential velocity. Once the droplets are entrained into the vortex environment they will absorb very little additional energy--save for a redistribution of the vortex velocity profile as particles near the center of the vortex migrate towards the outer reaches. This particle motion will be discussed in detail later. Due to the introduction of the water droplets, the fluid has assumed a two-phase character and may well suffer some change in apparent physical properties⁽¹⁾. Thus, an equivalent fluid viscosity, taking into account the influence of particles on the turbulent eddy viscosity, could be defined which, intuitively

suggests a retardation of the vortex, but physically may produce higher Reynolds numbers thus giving greater inertia to the two-phase flow and thus sustain the vortex life.

Another effect that may operate is the phenomenon of vortex bursting^(2,3). Due to the changes in physical properties and in the velocity profiles brought about by the water content, it is possible that vortex bursting would occur prematurely. As the understanding of vortex break-down is not well documented, very little can be said on this point at present, but stability theory does point out the significance of the local velocity gradients. It is, however, felt that a solution of the full Stokes-Navier equations will be necessary before this problem of vortex bursting can be fully appreciated⁽⁴⁾. It is urged that such a study be undertaken.

In the present report, some comments are made concerning the motion of non-interacting spherical particles in a vortex-type flow. The remarks are made in the context of an exploratory exposition rather than of a definitive study. Enough is said, however, to clarify the direction that future studies should take.

II. Preliminary Remarks

Before embarking upon the study in hand, a remark concerning the nature of fog is in order.

According to Tverkoi ⁽⁵⁾ the distribution of water droplet radius in a fog is characterized as in Figure 1. Typically, the mean particle has a radius $a=8 \times 10^{-4}$ cm and a mass $m = 2 \times 10^{-9}$ gm. It will be assumed in the present study that this median value is independent of the liquid water content present in the atmosphere. This may not be the case when the atmosphere is very near to its dew point.

When free to fall in a still atmosphere, this particle of mean diameter will have a motion described by the equation:

$$\frac{du}{dt} = g - \frac{6\pi a u \mu}{m} \quad (1)$$

In this equation, u is the velocity of the particle at time t . The fluid viscosity coefficient is denoted by μ . It is assumed in writing equation (1) that the particle experiences a retarding force given by Stokes' formula ⁽⁶⁾ for the slow motion of spherical bodies. Due to the smallness of a , surface tension dictates that the particle is essentially spherical. Equation (1) readily solves to show:

$$u = \frac{mg}{6\pi \mu a} \left\{ 1 - \exp \left[-\frac{6\pi \mu a t}{m} \right] \right\} \quad (2)$$

for a particle at rest at $t = 0$. This velocity is plotted as a function of time on Figure 2. Also included on this figure (for comparison with later studies) is the numerical solution to equation (1) obtained by means of a second order Runge-Kutta technique.

The terminal velocity for this mean diameter particle is 0.81 cm/sec giving a terminal Reynolds number of 0.01. The Stokes flow assumption is thus seen to be in order. The adequacy of the numerical solution is also evident from the plot of Figure 2 and this observation is pertinent to the subsequent work.

If the characteristic velocity $u_c = mg/6\pi\mu a$ and characteristic time $t_c = m/6\pi\mu a$ are defined then the Froude number $F_d \equiv u_c^2/ga$ and the Reynolds number $Re = u_c a \rho / \mu$ relate by the equation $Re = 9\rho F_d / 2\rho_w$. Equation (2) indicates that when $t = t_c$ then u is approximately 33% of u_c . Clearly, $t_c = F_d a^2 / Re \gamma$ so that for fixed physical conditions in the problem $t_c \sim a^2$.

III. Forces on a Spherical Particle

Figure 3 illustrates the coordinate system employed and the forces acting on the particle. Four forces will govern the motion of the particle. These include the gravitational force, the lift, drag and centrifugal forces. From the onset it is assumed that the fluid motion is laminar in nature so that the influence of a turbulent structure on the particle motion need not be considered. It is further assumed that the particle is spherical and of uniform composition. Because of this assumption, the particle motion may be treated as that of a point mass. The particle will experience no torque. Despite the fact that the local velocities inside the vortex may be considerable, it will still be assumed that the drag force experienced by the particle is given by Stokes' formula. As this equation is linear in the relative velocity between the sphere and the fluid, we are at liberty to resolve the drag force simply by using the velocity components. Possibly during the initial phase motion, when the relative velocity between fluid and particle is large, a more accurate expression for the drag force would be in order. However, in view of the large simplifications involved with the assumption of laminar motion, it is felt that such an elaboration would be misleading and inappropriate.

In this study it is assumed that the vortex axis is horizontal so that the gravitational force which acts downwards is perpendicular to the axis of the vortex.

Due to the curvature of the streamlines within the vortex there is a force induced on the particle which is directed towards the center of the vortex. Within first order accuracy⁽⁷⁾ this force is stated as $2\pi\rho a^3 w^2/r$. Here w is the relative velocity component in the azimuthal direction and r the distance of the particle from the vortex center. Finally, there is a centrifugal force acting radially outwards.

It is useful to compare the magnitudes of the various force components. To do this we note that the force components can be written:

$$\begin{aligned} F_z &= \lambda_1 (U-u) \\ F_r &= \lambda_1 (V-u) - \frac{\lambda_2}{r} (W-w)^2 + g \log \theta + \frac{w^2}{r} \\ F_g &= \lambda_1 (W-w) - g \end{aligned} \quad (3)$$

where

$$\lambda_1 = \frac{6\pi\mu a}{m} ; \lambda_2 = \frac{3}{2} \frac{\rho}{\rho_w}$$

For the standard size particle it follows that:

$$\lambda_1 = 1.2 \times 10^3 \text{ sec}^{-1}; \lambda_2 = 0.00559$$

Thus with the exception of positions very close to the vortex core ($r \ll 1$) the drag force predominates the motion. As a result it is expected that the particles will accelerate rapidly to the local flow condition and then move essentially with the streamlines of the flow.

It should also be pointed out that the particle mass only enters equations (3) via the drag term (λ_1). Here the relationship is an inverse one so that the heavier particle responds more slowly to the fluid motion.

Of course in the real flow where the motion is turbulent, these conclusions would need some modification.

IV. On a Model for the Vortex Flow

Let $\underline{w} \equiv (\xi, \eta, \zeta)$ be the vorticity vector in cylindrical coordinates (z, r, θ) . If in addition, the velocity components are (U, V, W) , then for a flow that is axially symmetric, given by the condition $\partial/\partial\theta \equiv 0$, it follows that (8):

$$\xi = \frac{1}{r} \frac{\partial W r}{\partial r}$$

$$\eta = -\frac{1}{r} \frac{\partial W r}{\partial z}$$

$$\zeta = \frac{\partial V}{\partial z} - \frac{\partial U}{\partial r}$$

The continuity requirement

$$\frac{\partial}{\partial z} (rU) + \frac{\partial}{\partial r} (rV) = 0$$

is satisfied by a streamfunction Ψ such that

$$U = \frac{1}{r} \frac{\partial \Psi}{\partial r} ; V = -\frac{1}{r} \frac{\partial \Psi}{\partial z}$$

The present model of a line vortex will admit variations in all three velocity components (U, V, W) but makes the assumption that the vorticity vanishes in the (r, z) plane. This demands that $\zeta \equiv 0$; then we find :

$$\frac{\partial^2 \lambda}{\partial r^2} + \frac{1}{r} \frac{\partial \lambda}{\partial r} + \frac{\partial^2 \lambda}{\partial z^2} - \frac{\lambda}{r^2} = 0 \quad (4)$$

where $\lambda = \psi/r$ denotes a modified streamfunction.

If it were further assumed that both ξ and η identically vanish, then it would follow that $Wr = \text{const}$; in conformity with the potential vortex flow. However, to remove the singularity on the axis, the result given by Lamb⁽⁹⁾

$$Wr = \frac{\Gamma}{2\pi} \left\{ 1 - \exp \left[\frac{-r^2 U_\infty}{4\nu z} \right] \right\} \quad (5)$$

has been adopted. Away from the vortex core, the flow is again essentially irrotational.

In this model of the vortex flow it is interesting to note that the flow in the (r,z) plane (i.e., the U,V velocity components) is completely uncoupled from the motion in the azimuthal direction. There is, then, no mechanism for the singularity on the axis, that is evident from the solution $w \sim 1/r$, to be removed by an axial velocity component along the axis⁽¹⁰⁾. This latter coupling is the province of the viscous nature of the flow: which is ignored herein.

It was suggested by Moore and Saffman⁽¹¹⁾ and confirmed experimentally⁽¹²⁾, at least for one value of Reynolds number, that the axial velocity along the core of a trailing laminar vortex can be expressed as:

$$\frac{U}{U_{\infty}} = \frac{0.28}{\sqrt{z/c}} \left\{ 1 + 8.57 \alpha^2 \sqrt{Re} \times 10^{-5} \right\} \quad (6)$$

where α is the wing incidence in degrees and c the wing chord length. Re is the Reynolds number of the flow based upon the wing chord and is sufficiently small for the vortex flow to be laminar. Hence, far from the wing it is expected that the axial velocity will decay as the inverse square root of the axial distance. The data⁽¹²⁾ shows that equation (6) loses its validity 'close' to the wing ($z/c < 100$).

In the present study, equation (4) is solved under an assumed initial distribution of axial velocity. The initial value to be adopted for u/U_{∞} is still in question but could be selected so as to satisfy equation (6) in the far field.

Using a Fourier-Bessel expansion, it is readily found that

$$U(r, z) = \sum_{n=1}^{\infty} A_n J_0(\mu_n r) \exp(-\mu_n z) \quad (7)$$

$$V(r, z) = \sum_{n=1}^{\infty} A_n J_1(\mu_n r) \exp(-\mu_n z)$$

$$\Psi(r, z) = r \sum_{n=1}^{\infty} \frac{A_n}{\mu_n} J_1(\mu_n r) \exp(-\mu_n z)$$

$$\Phi(r, z) = -\sum_{n=1}^{\infty} \frac{A_n}{\mu_n} J_1(\mu_n r) \exp(-\mu_n z)$$

where

$$A_n = \frac{2}{R [J_1(\mu_n R)]^2} \int_0^R r U_0(r) J_0(\mu_n r) dr$$

with μ_n the zeros of $J_0(\mu_n R) = 0$

and $U_0(r)$ is the assumed initial axial velocity distribution.

An expression like

$$U_0(r) = \frac{k_1}{1+k_2 r^2} \quad (8)$$

has the correct form for specified arbitrary constants k and k_1 .

The far field boundary conditions have been satisfied on $r = R$ where R can be several wing chord lengths in magnitude. Further comments concerning the solution of equation (4) are contained in the Appendix.

It is apparent from equation (7) that the axial velocity component along the axis dies away exponentially far from the origin of the vortex. This results contrasts with that contained in equation (6). A typical calculated distribution of axial velocity component along the axis is shown on Figure 4. Also plotted on this figure is the variation of axial velocity given by the approximation presented in the Appendix (equation A3).

The differences between the exponential and inverse square root decay rates is an indication of the fashion in which the solution is influenced by the inclusion of a viscous term.

Typical calculated axial and radial velocity components are shown in Figures 5 and 6 respectively.

V. An Indication of Particle Motion

As a prelude to embarking upon a discussion of the particle motion within the vortex, some orders of magnitude of the relevant terms can be written down.

First, a comparison can be made between the response of the particle to a change in the fluid velocity and the influence of gravity. For a particle subject only to a Stokes drag force, the equation of motion states

$$\frac{du}{dt} = \frac{6\pi\mu a}{m} (U_{\infty} - u) \quad (9)$$

if U_{∞} is the velocity of the fluid environment, then easily

$$\frac{u}{U_{\infty}} = 1 - \exp \left[-\frac{6\pi\mu a t}{m} \right]$$

which contains an identical exponential term to that in equation (2). Hence the response to gravity and changes in fluid velocity are enacted in the same time scale. This, of course, is clear from the physical nature of the situation as embodied in the respective differential equations.

It was seen from Figure 2 that the time required for $u/U_{\infty} \sim 0.99$ is $t = 4.6m/6\pi\mu a$ seconds or $t = 3.82 \times 10^{-3}$ sec for the standard particle. This suggests, as was noted above, that the general mode of motion will be for the particle, initially at rest, to very rapidly accelerate to the local fluid velocity and then progress with the fluid environment.

Hence, work done on the particle will predominantly be over the very short initial time period associated with the rapid acceleration phase of the motion. After this, the particle motion will deviate little from that of the fluid and the rate of work is extremely small.

To describe the motion in more detail, the equations of motion (3) were solved using a second order accurate Runge-Kutta integration procedure. The accuracy of this solution algorithm as applied to the current type of problem is shown in Figure 2 and was discussed previously. The velocity field U, V, W of the vortex was taken from the solution discussed above. Then with F_z, F_r and F_θ as in equation (3) we have

$$\begin{aligned}\frac{du}{dt} &= F_z \\ \frac{dv}{dt} &= F_r \\ \frac{dw}{dt} &= F_\theta\end{aligned}\tag{10}$$

$$u = \frac{dz}{dt} ; \quad v = \frac{dr}{dt} ; \quad w = r \frac{d\theta}{dt}$$

which give the particle position.

The computer program developed to solve these equations returns the particle position, velocity components, accelerations, and the total work done on the particle as a function of time. Figure 7 shows typical traces of the work done on the particle.

As expected from the tentative discussion of the nature of the motion, the rate of work is very large over the initial time period while the particle attains the fluid velocity. After this, very little work is performed. The traces of Figure 7 are for particles of the same mass and starting from different azimuthal positions at the same radial distance from the vortex axis. It is seen that, to first order, there is little influence of azimuthal starting position on the work done.

Figure 8 shows how the motion of particles of various masses progresses with time. All particles start at the same point in the vortex.

VI. An Estimate of the Total Energy

Let δW be the work done on a single particle. If there are N particles in a volume dV then the total work is

$$\begin{aligned} W &= \int_{dv} N \delta W r dr d\theta dz \\ &= 2\pi N U_{\infty} \int_0^R \int_0^t r \delta w dr dt \end{aligned}$$

as pointed out before, the integral in time need only be carried out over the initial acceleration phase of the motion.

In evaluating this work integral it has been assumed that the water content of the air is contained as a uniform distribution of mean sized water droplets. While not realistic, in light of the data presented in Figures 1 through 8, it is adequate for an order-of-magnitude calculation. It is also assumed that the work done is independent of azimuthal position (see Figure 7) so that one integration can be carried out analytically.

Using a typical value of liquid water content⁽⁵⁾ of 0.2 gm/m^3 , evaluation of the above integral gives $W/U_{\infty} = 0.0022 \text{ Newton sec.}$

For the vortex flow alone, it is found that the kinetic energy/free stream velocity is $0.418 \text{ Newton sec.}$

Hence, the energy change due to the acceleration of the particles is 0.53% of the total energy content of the vortex. It is felt that this is a small fraction of the vortex energy, and of itself, unlikely to strongly influence the vortex decay.

In view of this result, it is considered that if water content in a vortex does indeed influence vortex decay rates, then this must be enacted through some mechanism other than energy absorption during particle acceleration.

VII. Discussion

The study of particle motion within a vortex environment tended to indicate a general outward migration of particles from the center of the vortex. Very heavy particles tend to be more conscious of gravity and fall more directly through the vortex. Such particles are more rain-like and not the subject of the present study.

The two-phase nature of the flow implies changes in the physical properties of the fluid. These changes will manifest themselves as changes in the kinematic viscosity through both density and absolute viscosity variations. Methods of estimating these properties of two-phase mixtures are discussed in (1,16). Two conventional expressions from⁽¹⁾ are:

$$\mu_{tp} = x\mu_w + (1-x)\mu_g$$

$$\frac{1}{\rho_{tp}} = \frac{x}{\rho_w} + \frac{(1-x)}{\rho_g}$$

where x is the mass fraction of the liquid phase. Table 1 shows the predicted variation of μ_{tp} , ρ_{tp} and ν_{tp} with liquid mass fraction. The significant observation from this Table is that ν_{tp} decreases with an increase of water content.

The influence of viscosity on the vortex decay was given by Squire⁽¹³⁾, for turbulent flow and by McCormack et al⁽¹⁷⁾, for laminar flow. The latter result is:

$$\left(\frac{w_o}{w_{max}}\right)^2 = 1 + \left[\frac{4\nu_e \ln 2}{U_\infty r_h^2} \right] \quad (11)$$

Using values from an example calculation of Ref. (17), equation (11) produces the vortex decay effects shown in Table 2.

The calculations indicate that the molecular viscosity gives practically no vortex decay (in 20,000 ft) although what little trend there is gives a reduction in decay rate with increasing water content. The decay predicted by Squire's model for a turbulent vortex is shown in Figure 10. This figure demonstrates the effect of variations in eddy viscosity on the decay rate in an attempt to simulate the two-phase nature of the flow. Also shown in Figure (9) is experimental data⁽¹⁴⁾ for a vortex with the same circulation as that adopted in the theoretical study. The large miss-match in slope may be due to the fact (as noted above for laminar vortices) that the theory is only valid very far downstream.

Both theories predict large increases in vortex decay rate with increasing eddy viscosity, also, McCormack's experimental data shows a significantly larger rate of decay than that calculated for laminar flow which is a result of the eddy viscosity of turbulence. Hence the presence of fog can still, conceivably, result in a more rapid decay of the vortex if moisture serves to enlarge the eddy viscosity despite its effect on molecular viscosity. There is, however, very little information on the influence of water content on eddy viscosity and conflicting reports on two-phase mixing lengths give both larger and smaller values⁽¹⁾. It is known that the liquid phase does significantly influence mean velocity profiles and this indicates that it may exert a strong effect on the eddy viscosity. How the droplets would interact with the turbulence eddy structure is not known. This problem remains to be resolved.

The outward migration of droplets will have a tendency to cause a redistribution of the vortex velocity profile with the faster moving vortex core being retarded. It is possible that this could influence the mechanism of vortex bursting--particularly if the relationship between the axial and the azimuthal velocity components is disturbed. Also the redistribution of the droplets in the downstream direction results in a variation of the molecular viscosity across the vortex core. Further study is required to understand these effects on vortex bursting. Studies on vortex stability⁽⁴⁾ indicate that this phenomenon is closely related to the velocity gradients within the vortex. The above remarks strongly suggest that the water content in the vortex will go some way towards distorting the velocity profiles; and hence influence the flow stability.

Another energy exchange mechanism may also be at work. The moisture saturated air in the higher pressure outer flow could suffer condensation and thus be warmed, while the low pressure core flow would be cooled with attendant evaporation. This is an additional factor not included in the present model which may influence the vortex decay rate.

It is believed that a considerably greater insight into the above mentioned effect of fog on trailing vortex decay could be obtained from a more sophisticated analysis of the Navier-Stokes equation than that reported upon herein.

VIII. Concluding Comments

The above exploratory remarks concerning the nature of vortex motion in an atmosphere containing fog have brought to light certain facets of the situation that would merit further deliberation. Among these we may mention the following:

- 1) The results indicate that the kinetic energy transferred from the vortex to the particle during their acceleration is small. Thus water droplet acceleration is not likely to augment the rate of vortex decay.
- 2) This conclusion does not exhaust the possibility that other mechanisms of vortex decay will not be accelerated by the presence of fog in the fluid medium. The two-phase character of the fog for example can conceivably enlarge the eddy viscosity, even though at the same time the molecular viscosity is decreased, and thus increase the decay rates or precipitate early vortex bursting.
- 3) The method of the above analysis provides predictions of the velocity and particle trajectories subject to the assumptions of small particles and laminar inviscid vortex flow. It is suggested that the viscous term should be included and turbulence considered.
- 4) The somewhat inconclusive findings of this tentative study are not presented to imply that fog will not enhance vortex decay, but rather, to illustrate that many unanswered questions concerning this problem require further study for resolution.

Table 1

Predicted Variation of Physical Properties
With Mass Fraction of Water

x	ρ_{TP}	μ_{TP}	γ_{TP}
0.01	0.70	1.83×10^{-5}	2.63×10^{-5}
0.05	3.19	4.36	1.37
0.10	6.30	7.52	1.19
0.20	12.50	13.84	1.10

Table 2

Calculated Vortex Decay rates using McCormack's result (17) and Equation 11.

			$[1 - (\frac{W_0}{W_{\max}})^2] \times 10^4$			
x	γ_e	$\frac{4 \ln(2)}{U_\infty (u_{orh})} \gamma_e$	z = 5,000	10,000	15,000	20,000
0.01	2.63×10^{-5}	5.234×10^{-8}	-2.62	-5.23	-7.84	-10.46
0.05	1.37	2.73	-1.36	-2.73	-4.09	- 5.45
0.10	1.19	2.37	-1.19	-2.37	-3.56	- 4.74

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It would be most satisfactory if equation (4) of the main text were solved for a region of infinite radius. This solution can, of course, readily be written in terms of a Hankel transform, then if

$$A(\mu) = \int_0^{\infty} r U_0(r) J_0(\mu r) dr \quad A1$$

it follows that

$$U(R,z) = \int_0^{\infty} \mu A(\mu) J_0(\mu r) \exp(-\mu z) d\mu \quad A2$$

with similar expressions for v, ψ and ϕ .

If the form

$$U_0(r) = k_1 / (1 + k^2 r^2)$$

is taken for the initial profile, the equation A1 integrates to yield:

$$A(\mu) = \frac{k}{k_2} K_0(\mu/k)$$

with $K_0(\delta)$ the modified Bessel function.

Some algebra then gives an expression for $U(r,z)$ in terms of transcendental functions. Specifically we have:

$$U(r,z) = k_1 \sqrt{\pi} \sum_{i=0}^{\infty} \left(\frac{-k^2 r^2}{4} \right)^i \frac{\Gamma(\ell) \Gamma(\ell)}{\Gamma(\ell+1) \Gamma(\ell+\frac{1}{2})} F(\ell, \frac{1}{2}; \ell+\frac{1}{2}; \frac{kz-1}{kz+1})$$

with $\ell = 2(1+i)$

Here $\Gamma(\delta)$ and $F(n,m; \ell; \delta)$ denote the gamma and hypergeometric functions respectively.

It can easily be demonstrated, however, that this series only converges for a modest range of r and z and is not suitable for a general study.

Attempts to solve the above integrals by means of Abel transforms⁽¹⁵⁾ were not made.

On the axis equation A2 integrates directly to give:

$$U(0,z) = \frac{4 k_1}{3(1+kz)^2} F\left(2, \frac{1}{2}; \frac{5}{2}; \frac{kz-1}{kz+1}\right)$$

A series expansion than takes the form

$$U(0,z) \sim \frac{4k_1}{3(1+kz)^2} \left\{ 1 + \frac{2}{5} \frac{kz-1}{kz+1} + \dots \right\} \quad A3$$

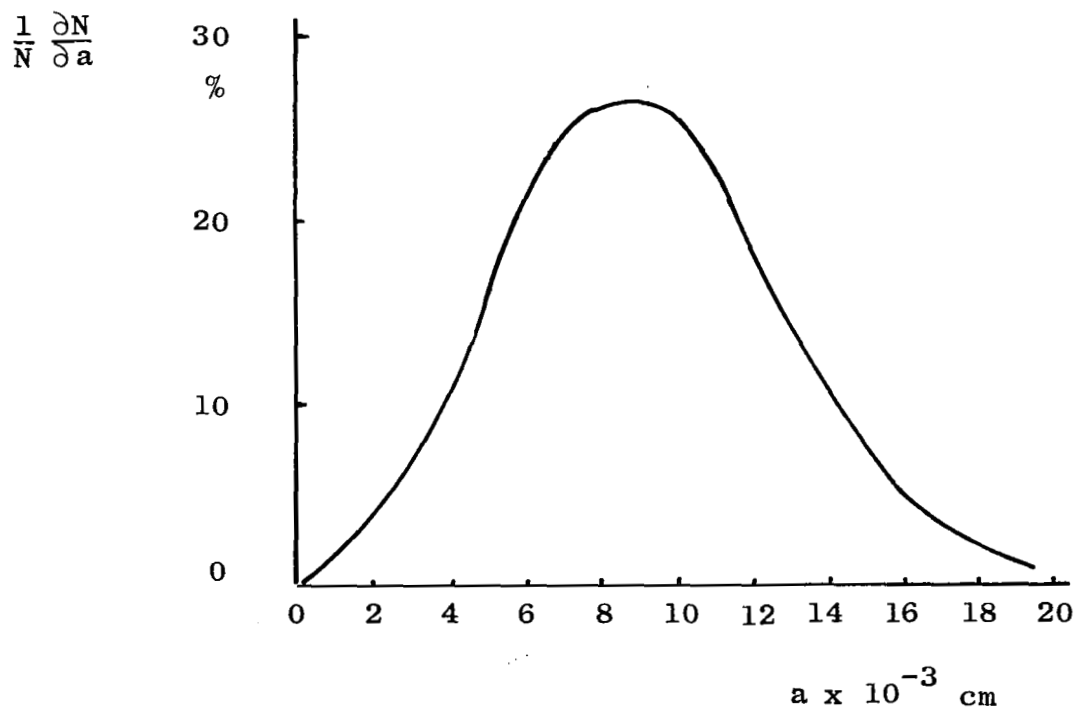


Figure 1. Distribution of Cloud Droplet Size From Ref. (5).

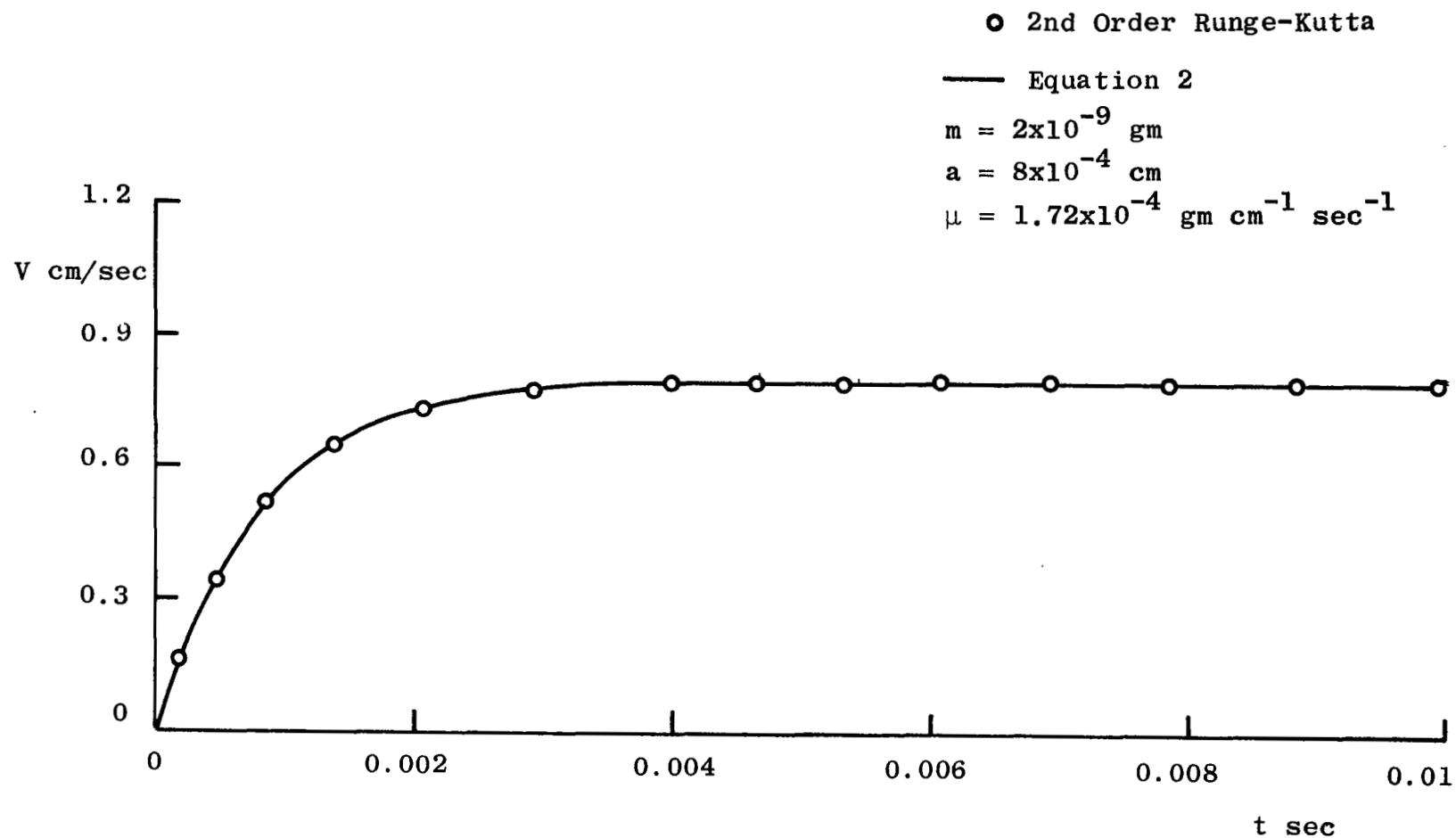


Figure 2. Motion of Mean Radius Particle Under Gravity and Stoke's Drag (Equation 2).

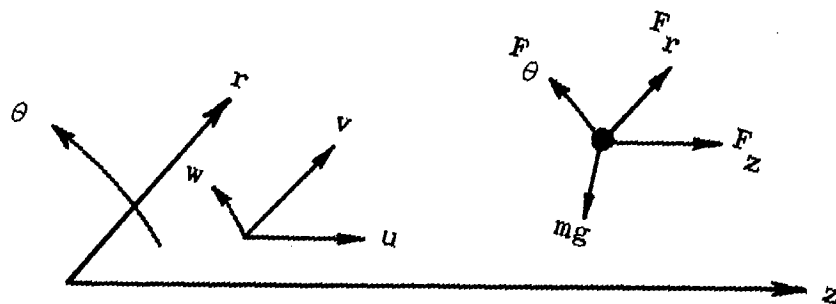


Figure 3. Coordinate System and Forces on a Particle

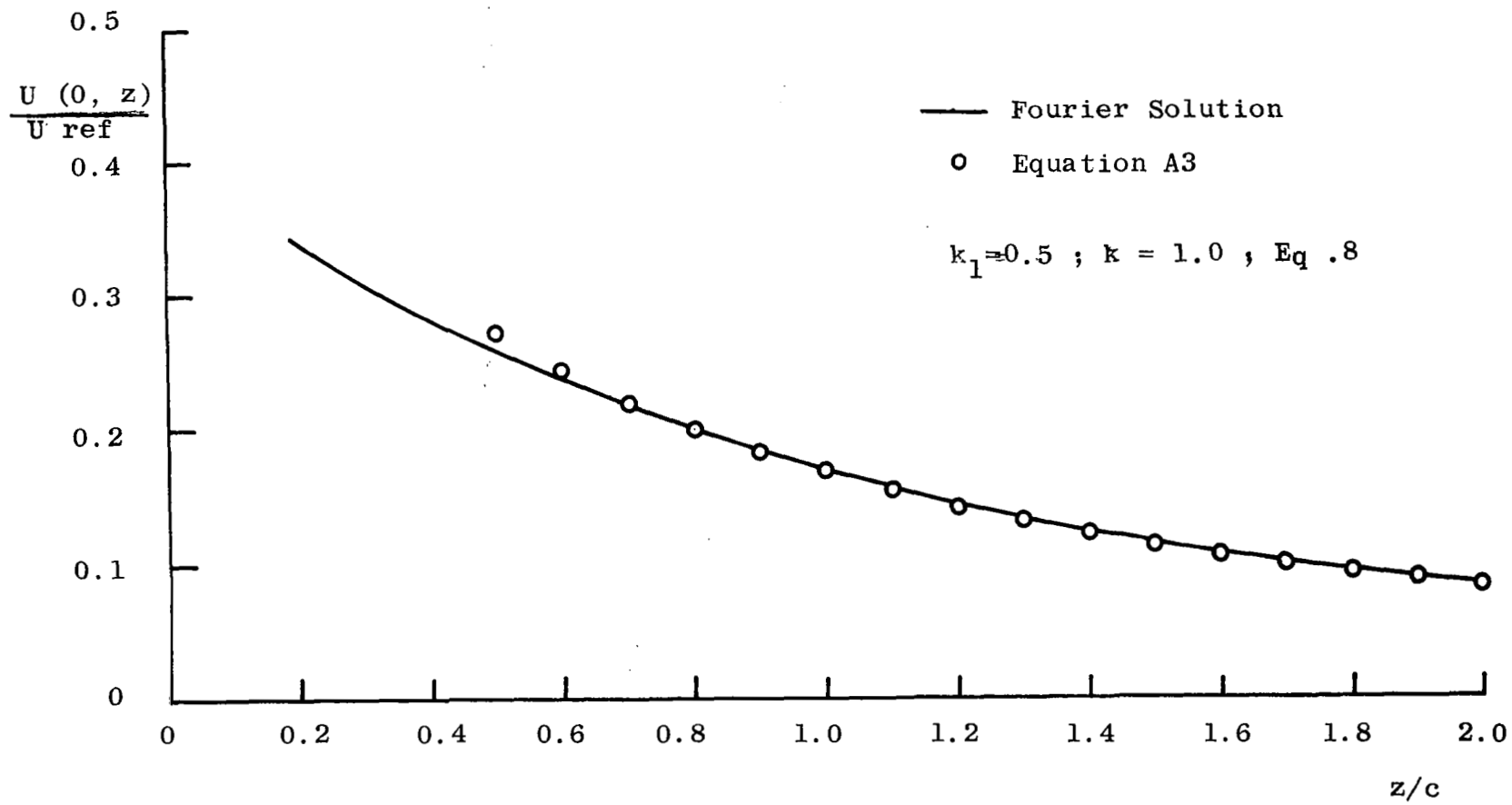


Figure 4. Axial Velocity Component On Axis of Vortex.

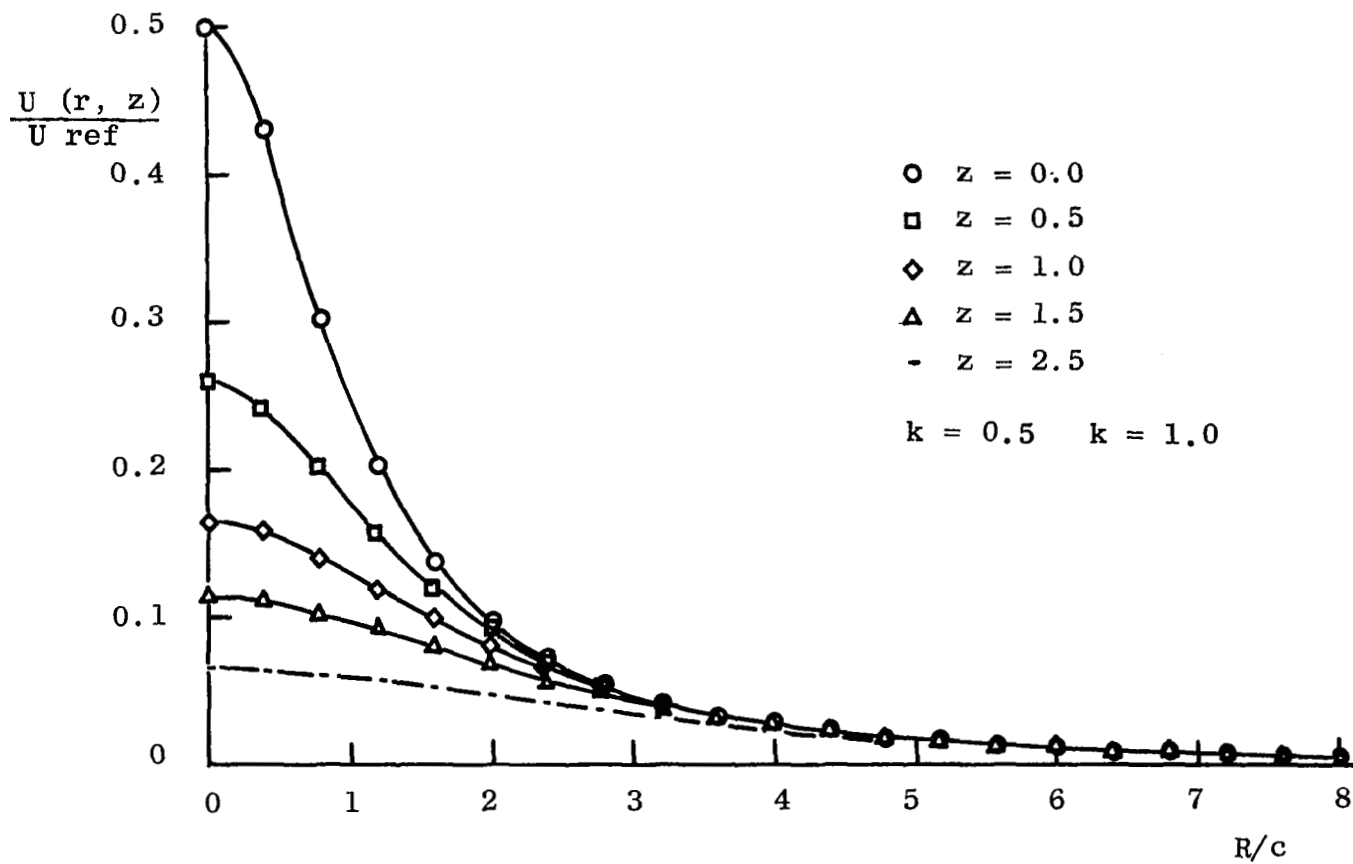


Figure 5. Distribution of Axial Velocity Component At Different Stations Along the Vortex.

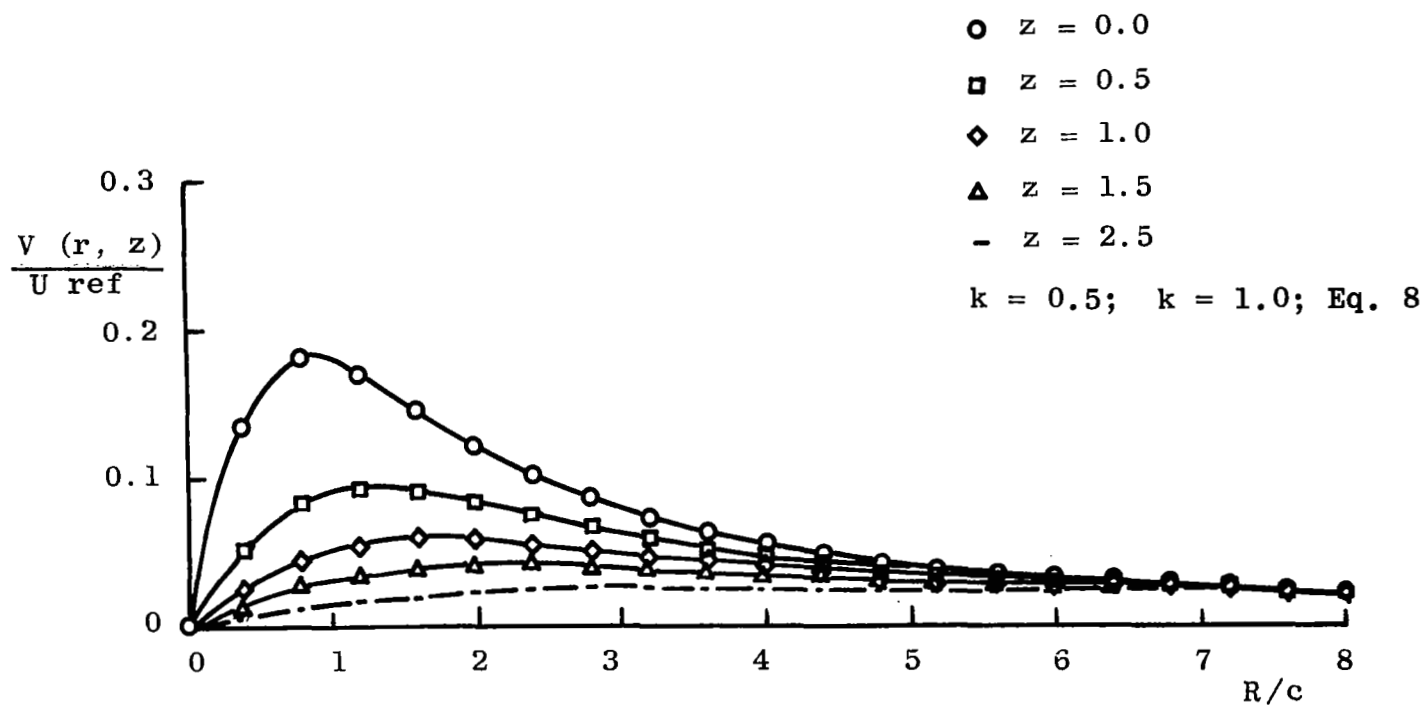


Figure 6. Distribution of Radial Velocity Component Along the Vortex.

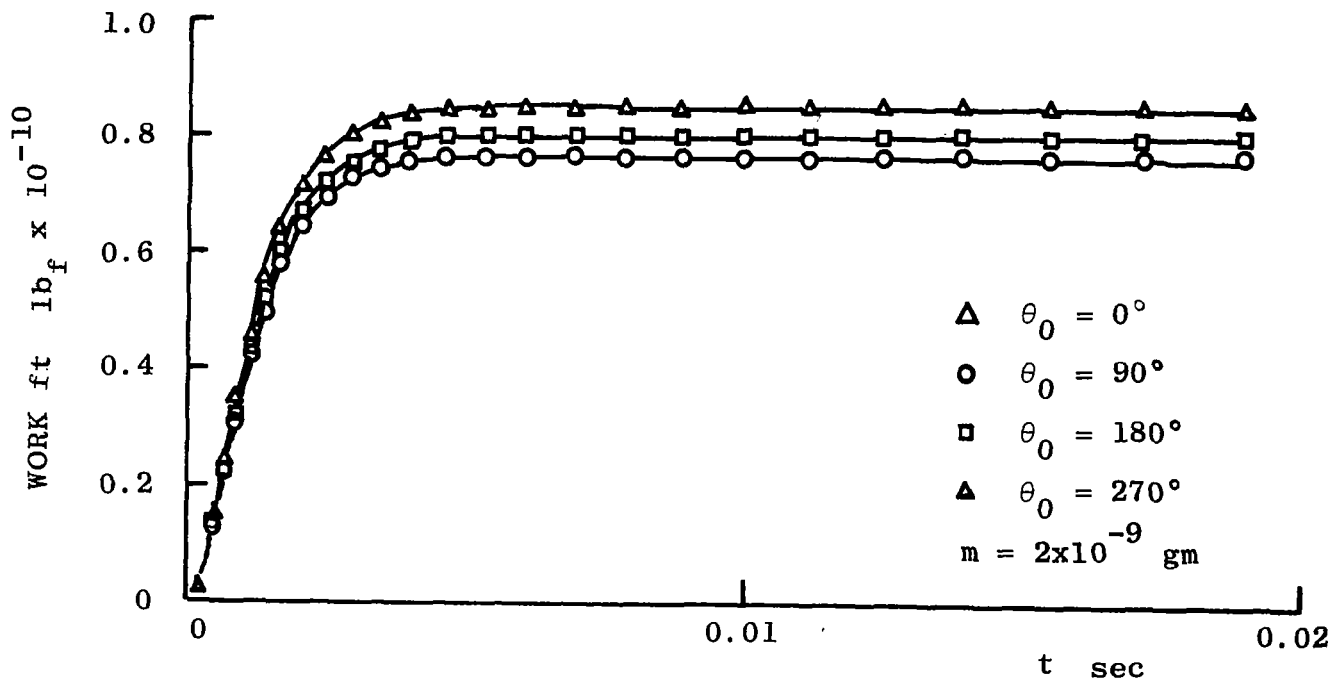


Figure 7. Work Done on Particle as a Function of Initial Asimuthal Position. For Vortex Discussed in Figures 4,5 and 6.

O $a = 100 \times 10^{-4} \text{ cm}$
 x $a = 20 \times 10^{-4} \text{ cm}$
 - - - $a = 1 \times 10^{-4} \text{ cm}$

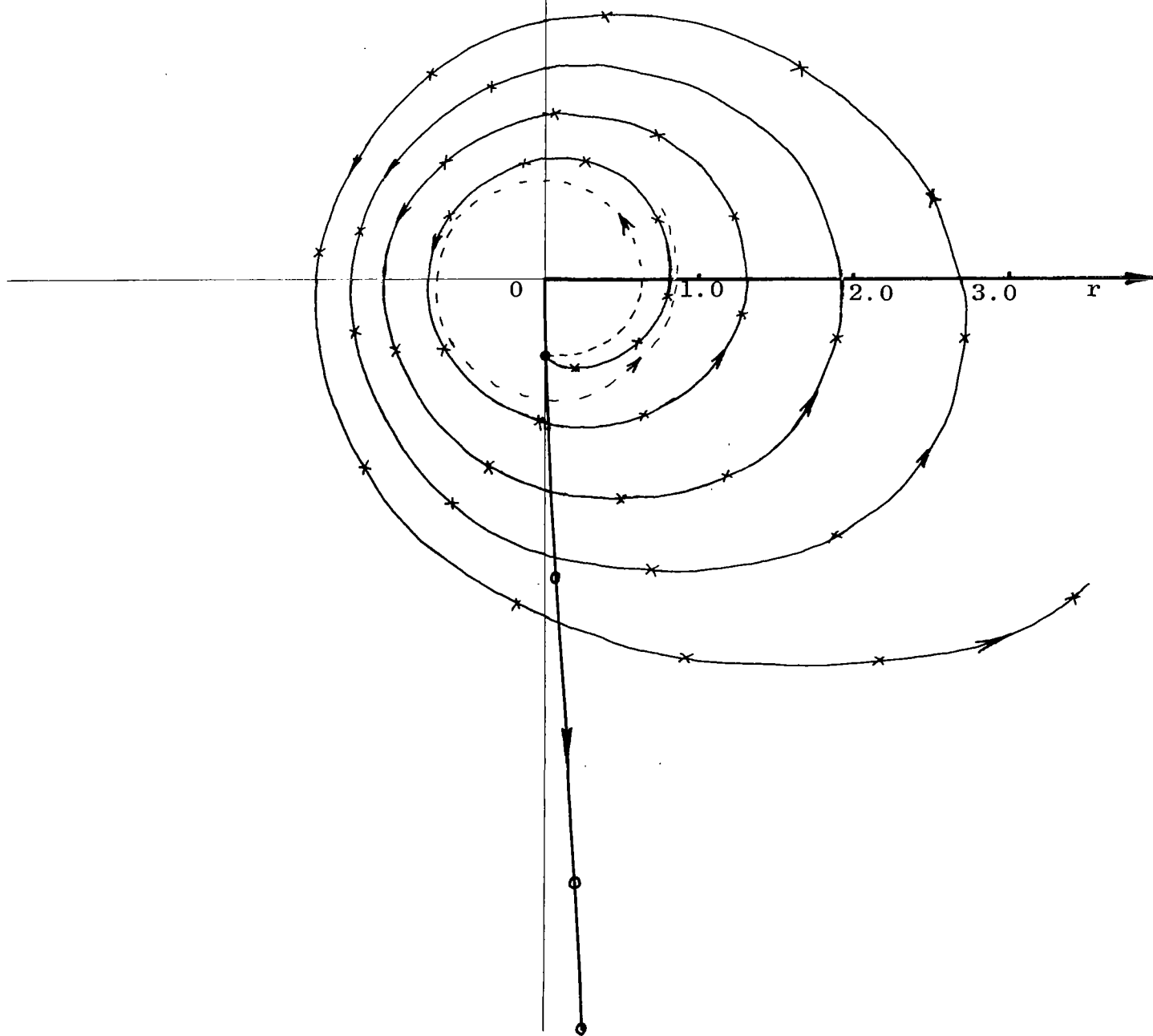


Figure 8. Particle Paths for Three Different Particle Masses.

○ Data Poppleton (Ref. 14)

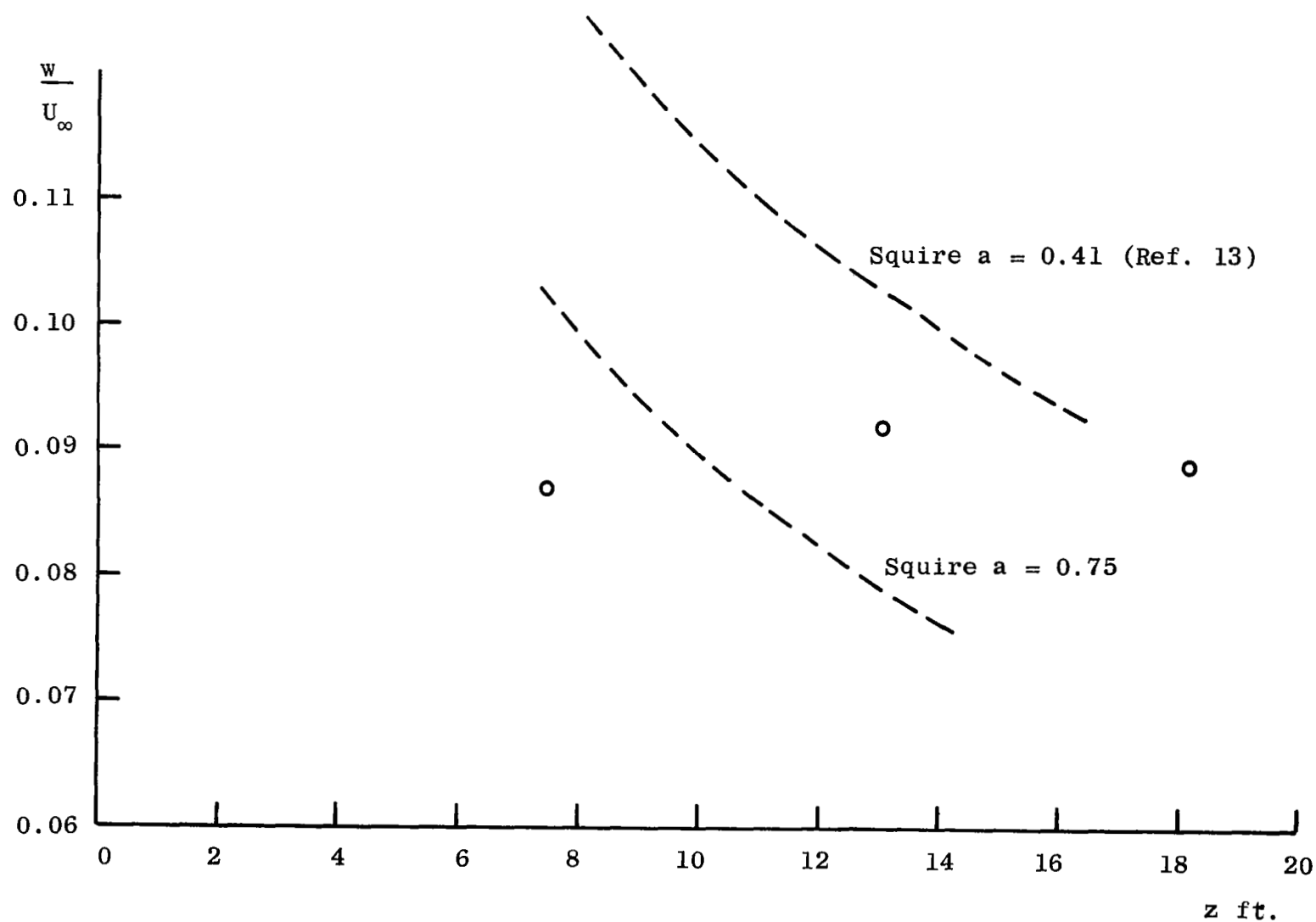


Figure 9. Decay of Peak Velocity Along Axis of Vortex.